Pre-class Warm-up!!!

How many ways can you think of to describe a curve in R^2 ?

a. ≥ 4

b. 3

c. 2

d. 1

e. 0

Quiz tomorrow: on the material from last week plus the week before that: 8.3, 6.1, 6.2, 1.4

7.3 Parametrized Surfaces.

We learn:

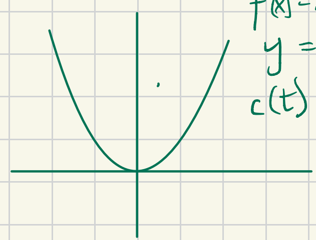
- what is a surface. It is like a sheet that bends
- we have 3 ways to describe a surface
- How to compute the tangent plane at a point from a parametrization.

- the graph of a function $f(x,y)$ like $f(x,y) = x^2 - y^2$
- the set of points (x,y,z) satisfying some equation, like $z = x^2 - y^2$
- by a parametrization

Review: we can sometimes describe a curve in 3 ways:

- the graph of a function, like the graph of f(x) = x^2
- The solution set of some equations, like y = x^2
- by a parametrization, like c(t) = (t, t^2)

We know how to convert between these and how to compute tangent lines to curves.



$f(x) = x^2$

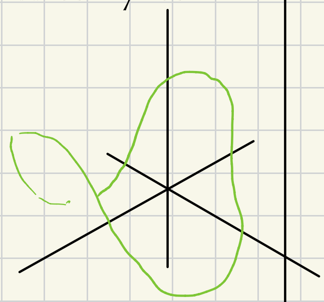
$y = x^2$

$c(t) = (t, t^2)$

Definition. A parametrization of a surface in R^3 is a function $\Phi : D \to \mathbb{R}^3$ where $D \subseteq \mathbb{R}^2$

Write $\Phi(u,v) = (x(u,v), y(u,v), z(u,v))$

Examples: The graph of f(x,y) = x^2 - y^2



- has a parametrization

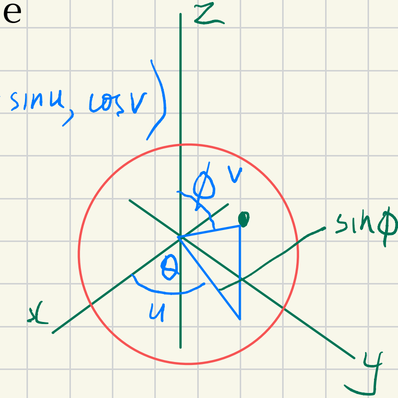
$\Phi(u,v) = (u, v, u^2 - v^2)$

- is the set of points satisfying an equation

$z = x^2 - y^2$

The surface itself is $\Phi(D)$.

- We should recognize parametrizations of a sphere, and of a cone

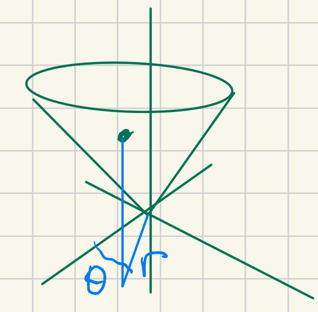


$\Phi(u,v) = (\sin v \cos u, \sin v \sin u, \cos v)$

$0 \leq u \leq 2\pi, \quad 0 \leq v \leq \pi$

$\Phi(u,v) = (u \cos v, u \sin v, u)$

$u \geq 0 \quad 0 \leq v < 2\pi$



$r = u$

$\theta = v$

Tangent vectors

Recall: for a path c : R -> R^3 the velocity vector c′ points tangentially to the curve.

$= \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$

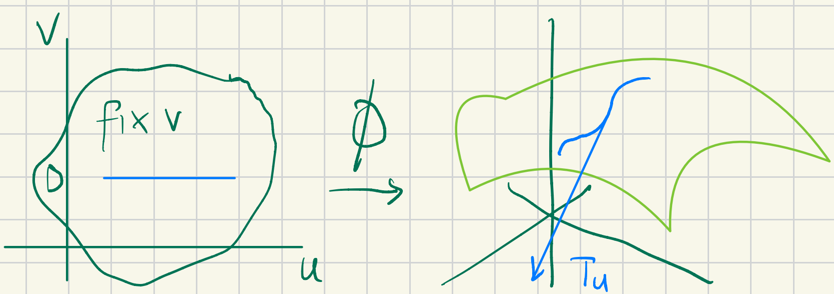
Given a parametrization of a surface

$$\Phi(u,v) = \big(x(u,v), y(u,v), z(u,v) \big)$$

we get two vectors tangential to the surface:

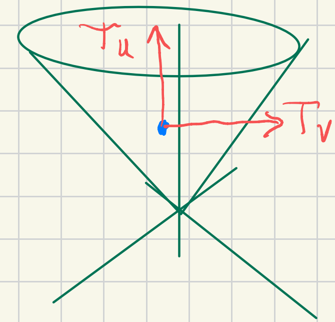
$$T_u = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right)$$

$$T_v = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right)$$



Example: the parametrization of the cone:

Phi (u,v) = (u cos(v), u sin(v), u),
$u \geq 0$, $0 \leq v \leq 2\pi$



$$T_u = (\cos v, \sin v, 1)$$

$$T_v = (-u \sin v, u \cos v, 0)$$

We get a normal vector

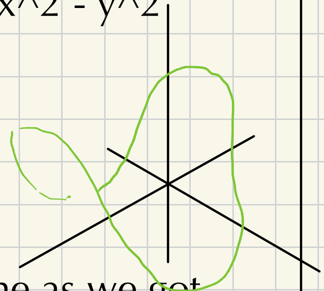
$$T_u \times T_v = (-u \cos v, -u \sin v, u)$$

Tangent plane:

$$-u \cos v (x - u \cos v) + \qquad + \qquad = 0$$

We always want the parametrization to be regular, meaning $T_u \times T_v \neq 0$ always.

Example: the graph of f(x,y) = x^2 - y^2
has parametrization



Phi (u,v) = (u, v, u^2 - v^2)

The tangent vectors are the same as we got previously.

$T_u = (1, 0, 2u) \qquad T_v = (0, 1, -2v).$

Question: are either of the following surfaces (that we just saw) the graphs of functions?

a. the sphere Yes No

b. the cone Yes No